

Pouliot Type Duality via a -Maximization

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We study four-dimensional $\mathcal{N} = 1$ $Spin(10)$ gauge theory with a single spinor and N_Q vectors at the superconformal fixed point via the electric-magnetic duality and a -maximization. When gauge invariant chiral primary operators hit the unitarity bounds, we find that the theory with no superpotential is identical to the one with some superpotential at the infrared fixed point. The auxiliary field method in the electric theory offers a satisfying description of the infrared fixed point, which is consistent with the better picture in the magnetic theory. In particular, it gives a clear description of the emergence of new massless degrees of freedom in the electric theory.

1. Introduction

Four-dimensional $\mathcal{N} = 1$ $Spin(10)$ gauge theory with one chiral superfield in the spinor representation and N_Q chiral superfields in the vector representation has rich and intriguing dynamics. In particular, it shows dynamical supersymmetry breaking [1,2] with no vectors and the electric-magnetic duality [3,4] for $7 \leq N_Q \leq 21$, the latter of which leads via the gauge symmetry breaking at some points in the moduli space to the duality [5] between chiral and vector-like gauge theories, as well as the one discussed in [6]. They all are so called the Pouliot-type dualities.

When the electric-magnetic duality is available, the dual pair is often found in the non-Abelian Coulomb phase [7]. Since the theory is at the non-trivial infrared fixed point, some exact results can be obtained by $\mathcal{N} = 1$ superconformal symmetry. In particular, the scaling dimension $D(\mathcal{O})$ of a gauge invariant chiral primary operator \mathcal{O} can be determined by the $U(1)_R$ charge $R(\mathcal{O})$ as

$$D(\mathcal{O}) = \frac{3}{2}R(\mathcal{O}).$$

The unitarity of representations of conformal symmetry requires the scaling dimension $D(\mathcal{O})$ of a scalar field \mathcal{O} to satisfy [8]

$$D(\mathcal{O}) \geq 1.$$

However, one sometimes encounters a gauge invariant chiral primary spinless operator \mathcal{O} which appears to satisfy the inequality $R(\mathcal{O}) < 2/3$. It has been discussed that such an operator \mathcal{O} decouples as a free field from the remaining interacting system, and an accidental $U(1)$ symmetry appears in the infrared to fix the $U(1)_R$ charge of the operator \mathcal{O} to $2/3$ [7,9,10].

One can see in the paper [5] that one of the examples is $Spin(7)$ gauge theory with $N_f = 7$ spinors Q^i ($i = 1, \dots, N_f$) and with no superpotential. Its dual or magnetic theory exists for $7 \leq N_f \leq 14$ and is given by $SU(N_f - 4)$ gauge theory with N_f antifundamentals \bar{q}_i and a single symmetric tensor s , along with gauge singlets M^{ij} , which can be identified with $Q^i Q^j$ in the electric theory. The superpotential W_{mag} of the magnetic theory is given by

$$W_{\text{mag}} = \frac{\tilde{h}}{\tilde{\mu}^2} M^{ij} \bar{q}_i s \bar{q}_j + \frac{1}{\tilde{\mu}^{N_f-7}} \det s,$$

where $\tilde{\mu}$ is a dimensionful parameter to give the correct mass dimension to M^{ij} , and the dimensionless parameter \tilde{h} shows up because we assume that the field M^{ij} has the canonical kinetic term.

As discussed in [5], since the $U(1)_R$ charge of the spinors Q^i is given by $1 - (5/N_f)$, the gauge invariant operator M^{ij} appears to violate the unitarity bound for $N_f = 7$ and therefore propagates as a free field at the infrared fixed point. From the viewpoint of the magnetic theory, it implies that the parameter \tilde{h} in the superpotential W_{mag} goes to zero in the infrared. Then, the F-term condition

$$\frac{\tilde{h}}{\tilde{\mu}^2} \bar{q}_i s \bar{q}_j = \frac{\partial W_{\text{mag}}}{\partial M^{ij}} = 0$$

doesn't impose any constraints on the gauge invariant operators $N_{ij} = \bar{q}_i s \bar{q}_j$, and the new massless degrees of freedom N_{ij} show up in the low-energy spectrum. One can easily see that the resulting magnetic theory at the fixed point has a different electric dual from the original electric theory with no superpotential.

In fact, its electric dual is the same as the original electric theory except that it has the non-zero superpotential

$$W_{\text{ele}} = \frac{1}{\mu} N_{ij} Q^i Q^j,$$

along with free singlets M^{ij} . Thus, one can conclude that these two electric theories are identical at the infrared fixed point. It also means that the original dual pair, consisting of the $Spin(7)$ gauge theory with no superpotential and the magnetic theory with the superpotential W_{mag} , flows into another dual pair, consisting of the $Spin(7)$ theory with the superpotential W_{ele} and the magnetic dual with vanishing \tilde{h} in the superpotential in the deepest infrared.

From the point of view on the electric side, the same dynamics can be captured by the auxiliary field method¹. In the original $Spin(7)$ gauge theory, turning on the superpotential

$$W = \frac{1}{\mu} N_{ij} (Q^i Q^j - h M^{ij}),$$

where the auxiliary fields M^{ij} and the Lagrange multipliers N_{ij} are introduced with the parameter h , does not change the original theory at all, as far as h is non-zero. The equations of motion give the constraints

$$Q^i Q^j = h M^{ij}, \quad h N_{ij} = 0.$$

¹ This method has been employed in [11] to give a more elaborate argument about the prescription in [10] to give the trial a -function when gauge invariant operators hit the unitarity bounds. Here and also below, we extend the idea for the models under consideration in this paper.

In the case $N_f = 7$, since the $U(1)_R$ charge of the operator M^{ij} hits the unitarity bound, the interaction of the field M^{ij} vanishes. Therefore, the coupling h goes to zero in the infrared to be consistent with the magnetic picture. When $h = 0$, it apparently becomes a different theory from the original one and gives the above-mentioned $Spin(7)$ theory with the superpotential W_{ele} . In addition, the F-term condition means that the directions $Q^i Q^j$ are redundant, but the new degrees of freedom N_{ij} are gained. Thus, the auxiliary field method gives a satisfying description of gauge invariant operators hitting the unitarity bounds on the electric side.

So far, we have seen that the data of the $U(1)_R$ charges is very powerful to uncover the rich infrared dynamics at the superconformal fixed points. However, when a superconformal theory has global $U(1)$ symmetries other than $U(1)_R$ symmetry, there *a priori* exists difficulty in finding which linear combination of $U(1)$ symmetries belongs to the superconformal algebra, as in our $Spin(10)$ gauge theory. This is the place that *a*-maximization [12] comes to the rescue. The application of the *a*-maximization method to our $Spin(10)$ theory is one of the main points of this paper, where we have one flavor $U(1)$ symmetry other than the $U(1)_R$ symmetry.

Let us suppose there are several non-anomalous flavor $U(1)$ symmetries other than $U(1)_\lambda$ symmetry. The latter transforms gaugino λ_α as $\lambda_\alpha \rightarrow e^{i\theta} \lambda_\alpha$ and, if necessary to make it non-anomalous, the other fields in an appropriate way², while the former leaves the gaugino intact. The superconformal $U(1)_R$ symmetry, if it isn't an accidental symmetry in the infrared, should be given by a linear combination of these $U(1)$ symmetries. Therefore, the $U(1)_R$ charge R_I of an operator Φ_I in the infrared may be given by the flavor $U(1)$ charges F^i_I and the $U(1)_\lambda$ charge Λ_I as $R_I(s) = \Lambda_I + \sum_i s^i F^i_I$ with fixed real numbers s^i , where the index i labels the $U(1)$ symmetries other than the $U(1)_\lambda$ symmetry. Since the $U(1)_R$ current and the energy-momentum tensor belong to the same superconformal multiplet, the anomaly coefficient in the three-point function with one of the flavor $U(1)$ currents inserted at one vertex and the $U(1)_R$ current at each of the two remaining vertices is related to the one with the same flavor $U(1)$ current at one vertex and the energy-momentum tensor at each of the remaining two vertices in the corresponding triangle Feynman diagrams. Therefore, as Intriligator and Wecht discussed in the seminal paper [12], the above parameters s^i giving the superconformal $U(1)_R$ symmetry are required to be the solution to

$$\frac{\partial}{\partial s^i} a(s) = 0, \quad \frac{\partial^2}{\partial s^i \partial s^j} a(s) < 0, \quad (1.1)$$

² Here, just for simplicity, we assume that the gauge group is simple.

for all i, j , where the function $a(s)$ is given [13,14,12] in the asymptotically free gauge theories via the 't Hooft anomaly matching condition [15] by³

$$a(s) = \sum_{A \in UV} \left[3(R_A(s) - 1)^3 - (R_A(s) - 1) \right] \quad (1.2)$$

in terms of the $U(1)_R$ charges $R_A(s)$ of the fundamental particles ϕ_A at high energy. The latter condition means that all the eigenvalues of the matrix on the left hand side should be negative. Since the matrix is related to the two-point functions of the $U(1)$ currents, the unitarity requires the latter condition [12,16].

However, if there exists the region of the parameter space spanned by $\{s^i\}$ where the $U(1)_R$ charge $R_I(s)$ of an operator Φ_I seems to violate the unitarity bound, one needs to subtract the contribution of Φ_I from the function $a(s)$, since it becomes a free field with the fixed $U(1)_R$ charge $R_I = 2/3$ at the point of the parameter space to maintain the unitarity of the theory⁴ [10]. Therefore, in different regions with different gauge invariant operators hitting the unitarity bounds, one needs to improve the a-function $a(s)$ and examine the existence of a local maximum in each of the regions. Following the prescription of the paper⁵ [10], one need to modify the a-function $a(s)$ as

$$a(s) - F(R_I) + F_0, \quad F(x) = 3(x - 1)^3 - (x - 1), \quad (1.3)$$

if a field Φ_I decouples to be free at some points of the parameter space $\{s^i\}$. Furthermore, since $R_I = 2/3$ on the boundary of these two regions, the function $a(s)$ and its first derivative with respect to the parameters s^i have the same values as $a(s) - F(R_I) + F_0$ and its first derivative, respectively. Thus, one obtains a continuous function on the whole parameter space $\{s^i\}$, which is not necessarily a third order polynomial in the parameters s^i as a whole. This suggests that we could find more than one local maximum of the whole function $a(s)$.

³ In this paper, we are not interested in the overall normalization of the a -function and will thus omit it. In order to get the conventionally normalized a -function, one needs to multiply $3/32$ with the function $a(s)$ of this paper.

⁴ Indeed, as discussed in [12], when Φ_I becomes free, an accidental $U(1)_\Phi$ symmetry appears and enables us to fix the $U(1)_R$ charge of Φ_I to $2/3$ via a -maximization, while keeping the $U(1)_R$ charge of the other operators unchanged.

⁵ We will see below that the prescription is consistent with the electric-magnetic duality, when the hitting operators are elementary fields in the magnetic theory.

Within a region with the same content of decoupling gauge invariant operators in the whole parameter space $\{s^i\}$, one can find at most a single local maximum, but in another region, one could obtain another local maximum, where one should find the different content of interacting massless gauge invariant operators. It may suggest that one could find more than one local maximum over the whole parameter space to lose definitive results on which linear combination of the $U(1)$ symmetries is the superconformal $U(1)_R$ symmetry. The weaker version of the very recent proposal in the paper [17] could however be a way out of this problem. It says that “*the correct IR phase is the one with the larger value of the conformal anomaly a* ”. It would thus be very interesting to find models with more than one local maximum of the function $a(s)$ and to study the renormalization group flow in the models.

In this paper, we will find a local maximum of the whole function $a(s)$ for $7 \leq N_Q \leq 21$. However, we haven’t completely confirmed that it is a unique local maximum, mainly due to difficulty in identifying the massless spectrum in the infrared and also due to the lack of our understanding about a -maximization applied to a gauge invariant operator in a non-trivial representation of the Lorentz group, as we will discuss later. We therefore will have to leave this question to the future. The problem, even if it really exists, doesn’t affect our results on the local maximum in this paper, except for the uniqueness of it.

The identification of the superconformal $U(1)_R$ symmetry enables us to know which gauge invariant chiral primary operators hit the unitarity bounds. We will discuss the renormalization group flow of the dual pair of our theories into another dual pair and the auxiliary field method of the electric theory. We will also check the consistency of our results by finding which operators as perturbation in the superpotential are irrelevant at the superconformal fixed point. Since the operators hitting the unitarity bounds are free, the couplings to them in the superpotential go to zero. Therefore, the corresponding perturbations must be irrelevant at the infrared fixed point.

To keep this paper within reasonable length, we will report our results for $Spin(10)$ gauge theory with more than one spinor in another paper [18].

This paper is organized as follows: in section two, we will give a brief review on the electric-magnetic duality [3,4] in the $Spin(10)$ gauge theory. In section three, the whole a -function will be constructed and the $U(1)_R$ charges are determined via the a -maximization method. Section four is devoted to summary and discussion, where the flow of our dual pair into another dual pair and the above auxiliary field method will be discussed.

2. The Electric-Magnetic Duality in the $Spin(10)$ Gauge Theories

Let us begin with a brief review of the electric-magnetic duality [3,4] of the theory we will consider in this paper. We study four-dimensional $\mathcal{N} = 1$ supersymmetric $Spin(10)$ gauge theory with a single spinor Ψ and N_Q vectors Q^i ($i = 1, \dots, N_Q$) and with no superpotential. Under the non-anomalous global symmetries $SU(N_Q) \times U(1) \times U(1)_\lambda$, the matter superfields Ψ and Q^i transform as $(1, N_Q, 0)$ and $(N_Q, -2, \frac{N_Q-6}{N_Q})$, respectively, as our convention.

It is believed [3,4] that it is in the non-Abelian Coulomb phase for $7 \leq N_Q \leq 21$, where it is asymptotically free and has the dual or magnetic description. The magnetic theory is given by $SU(N_Q - 5)$ gauge theory with N_Q antifundamentals \bar{q}_i , a single fundamental q , a symmetric tensor s and singlets M^{ij} and Y^i with the superpotential

$$W_{\text{mag}} = \frac{\tilde{h}}{\tilde{\mu}^2} M^{ij} \bar{q}_i s \bar{q}_j + \frac{\tilde{h}'}{\tilde{\mu}^2} Y^i q \bar{q}_i + \frac{1}{\tilde{\mu}^{N_Q-8}} \det s. \quad (2.1)$$

Only for $N_Q = 7$, one have the additional term

$$\frac{\tilde{h}''}{\tilde{\mu}^{15}} \epsilon_{i_1 \dots i_7} \epsilon_{j_1 \dots j_7} M^{i_1 j_1} \dots M^{i_6 j_6} Y^{i_7} Y^{j_7}$$

in the above superpotential W_{mag} , as discussed in [3,4].

Thus, at the infrared fixed point, it is an $\mathcal{N} = 1$ superconformal field theory with the superconformal $U(1)_R$ symmetry out of linear combinations of the $U(1) \times U(1)_\lambda$ symmetries. The $U(1)_R$ charges of the matter fields should satisfy the Adler-Bell-Jackiw $U(1)$ anomaly cancellation condition [19,20]

$$2R(\Psi) = -N_Q R(Q) + (N_Q - 6),$$

and thus turn out to be given by $R(Q) = \frac{N_Q-6}{N_Q} - 2x$, $R(\Psi) = N_Q x$, respectively, for a fixed real number x . According to a -maximization [12], the fixed real number x or equivalently the $U(1)_R$ charge $R \equiv R(Q)$ of the vectors Q^i must be the solution to the conditions (1.1). If there aren't any gauge invariant operators hitting the unitarity bounds, one can invoke the 't Hooft anomaly matching condition [15] to give the a -function in terms of the elementary fields as

$$a_0(R) = 90 + 16F[R(\Psi)] + 10N_Q F[R(Q)], \quad (2.2)$$

where the function $F(x)$ was defined in (1.3). The first term on the right hand side comes from the contribution of the gaugino, which are forty-five Weyl spinors of charge one with respect to the $U(1)_R$ symmetry, thus giving $45 \times [3R(\lambda)^3 - R(\lambda)] = 90$.

In this theory, there are several gauge invariant chiral operators⁶

$$\begin{aligned} M^{ij} &= Q^i Q^j, & Y^i &= Q^i \Psi \Gamma^{(1)} \Psi, & B^{i_1 \dots i_5} &= Q^{i_1} \dots Q^{i_5} \Psi \Gamma^{(5)} \Psi, \\ D_0^{i_1 \dots i_6} &= Q^{i_1} \dots Q^{i_6} W_\alpha W^\alpha, & D_{1\alpha}^{i_1 \dots i_8} &= Q^{i_1} \dots Q^{i_8} W_\alpha, & D_2^{i_1 \dots i_{10}} &= Q^{i_1} \dots Q^{i_{10}}, \\ E_0^{i_1 \dots i_5} &= Q^{i_1} \dots Q^{i_5} \Psi \Gamma^{(1)} \Psi W_\alpha W^\alpha, & E_{1\alpha}^{i_1 \dots i_7} &= Q^{i_1} \dots Q^{i_7} \Psi \Gamma^{(1)} \Psi W_\alpha, \\ E_2^{i_1 \dots i_9} &= Q^{i_1} \dots Q^{i_9} \Psi \Gamma^{(1)} \Psi, \end{aligned}$$

where $\Psi \Gamma^{(1)} \Psi$ and $\Psi \Gamma^{(5)} \Psi$ are bilinear combinations out of the spinor Ψ together with the gamma matrices of the gauge group $Spin(10)$ and transform in the vector representation and as a fifth rank antisymmetric tensor, respectively. The chiral superfield W_α is the $Spin(10)$ gauge superspace field strength in the adjoint representation of the gauge group. Note that depending upon the number N_Q of the vectors, some of the gauge invariant operators aren't available, as illustrated in Figure 1.

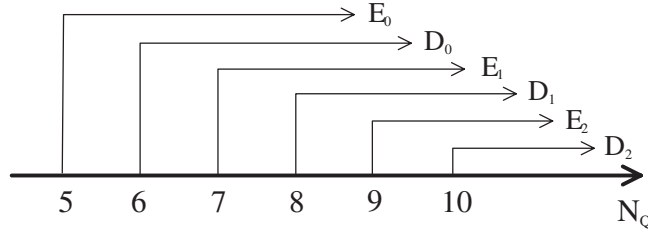


Figure 1: The number N_Q of the vectors Q^i where the gauge invariant operators D_n and E_n exist.

In the magnetic theory, the gauge invariant fields M^{ij} and Y^i are introduced as elementary fields, while the other gauge invariant operators can be constructed out of the antifundamentals \bar{q}_i , the fundamental q , the symmetric tensor s , and the dual gauge

⁶ We omit for simplicity explicit indices of the gauge group $Spin(10)$, which we assume are obvious in context. In particular, the gauge invariant operators are built out of the elementary chiral superfields with the invariant Kronecker deltas and the invariant tenth rank antisymmetric tensors.

Gauge Invariant Operators \mathcal{O}	the $U(1)$ charge	the $U(1)_R$ charge $R(\mathcal{O})$
$M \sim Q^2$	-4	$2R$
$Y \sim Q\Psi^2$	$2N_Q - 2$	$N_Q - 6 - (N_Q - 1)R$
$B \sim Q^5\Psi^2 \sim \bar{q}^{N_Q-5}$	$2N_Q - 10$	$N_Q - 6 - (N_Q - 5)R$
$D_n \sim Q^{6+2n}W^{2-n} \sim \bar{q}^{N_Q-6-2n}s^{N_Q-6-n}\widetilde{W}^n q$	$-4n - 12$	$(2n + 6)R - (n - 2)$
$E_n \sim Q^{5+2n}\Psi^2W^{2-n} \sim \bar{q}^{N_Q-5-2n}s^{N_Q-5-n}\widetilde{W}^n$	$-4n + 2N_Q - 10$	$(2n - N_Q + 5)R - (n - N_Q + 4)$

Table 1: The charges of the gauge invariant operators with respect to the $U(1) \times U(1)_R$ symmetry.

superspace field strength \widetilde{W}_α as

$$\begin{aligned}
(*B)_{j_1 \dots j_{N_Q-5}} &\sim \bar{q}_{j_1} \dots \bar{q}_{j_{N_Q-5}}, \\
(*D_0)_{j_1 \dots j_{N_Q-6}} &\sim q(s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-6}}), \quad (*D_{1\alpha})_{j_1 \dots j_{N_Q-8}} \sim q(s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-8}}) (\widetilde{W}_\alpha s), \\
(*D_2)_{j_1 \dots j_{N_Q-10}} &\sim q(s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-10}}) (\widetilde{W}_\alpha s) (\widetilde{W}^\alpha s), \\
(*E_0)_{j_1 \dots j_{N_Q-5}} &\sim (s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-5}}), \quad (*E_{1\alpha})_{j_1 \dots j_{N_Q-7}} \sim (s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-7}}) (\widetilde{W}_\alpha s), \\
(*E_2)_{j_1 \dots j_{N_Q-9}} &\sim (s\bar{q}_{j_1}) \dots (s\bar{q}_{j_{N_Q-9}}) (\widetilde{W}_\alpha s) (\widetilde{W}^\alpha s),
\end{aligned}$$

where the operation $*$ on the gauge invariant operators denotes the Hodge duality with respect to the flavor $SU(N_Q)$ symmetry. It is interesting to note that the classical moduli D_2 and E_2 in the electric theory are given by the gauge invariant operators containing the dual gaugino superfield⁷ \widetilde{W}_α . Besides the above operators, in the magnetic theory, there are other gauge invariant operators such as $N_{ij} = \bar{q}_i s \bar{q}_j$, $\det s$, $q\bar{q}_i$. They, however, are redundant, due to the F-term condition from the superpotential W_{mag} .

It follows from the superpotential W_{mag} that the elementary fields \bar{q}_i , q , and s in the magnetic theory have the charges $\left(2, \frac{N_Q-6}{N_Q-5} - R\right)$, $\left(-2N_Q, N_Q(R-1) + \frac{7N_Q-34}{N_Q-5}\right)$, and $\left(0, \frac{2}{N_Q-5}\right)$, respectively, with respect to the $U(1) \times U(1)_R$ symmetry. These charges satisfy the Adler-Bell-Jackiw $U(1)$ anomaly cancellation condition [19,20] and further are

⁷ The operators D_0 and E_0 can be rewritten as $(q\bar{q}) \cdot B \cdot M$ and $\det s \cdot B$, respectively in the magnetic theory. The operators $q\bar{q}_j$ and $\det s$ are redundant, as will be seen below. Therefore, they should be redundant at least for $7 \leq N_Q \leq 21$.

consistent with the mapping of the gauge invariant operators between the electric side and the magnetic one as shown in Table 1.

For later convenience, we introduce another dual pair, into which we will see below that the previous dual pair flows in the infrared for $7 \leq N_Q \leq 9$. When in the electric theory, turning on the superpotential

$$W_{\text{ele}} = \frac{1}{\mu^2} N_{ij} Q^i Q^j,$$

one finds that the moduli M^{ij} are eliminated because of the F-term condition

$$\frac{\partial}{\partial N_{ij}} W_{\text{ele}} = \frac{1}{\mu^2} Q^i Q^j = 0,$$

and instead that the new moduli N_{ij} show up. It can be seen in the magnetic theory that the first term $M^{ij} \bar{q}_i s \bar{q}_j$ in the superpotential W_{mag} has to be turned off to decouple the gauge singlets M^{ij} . The new F-term condition from the magnetic superpotential doesn't impose any constraints on the gauge invariant operator $N_{ij} = \bar{q}_i s \bar{q}_j$. Although the use of N_{ij} seems the abuse of the notation, the two on the both sides are in the same representation

$$N_{ij} : (\overline{\square}, 4, 2 - R(M) = 2(1 - R))$$

of the global symmetries $SU(N_Q) \times U(1) \times U(1)_R$ and can thus be identified. The field N_{ij} will play an important role, when the gauge invariant operator M^{ij} hits the unitarity bound in the original $Spin(10)$ theory. The electric $Spin(10)$ theory with the superpotential W_{ele} therefore is dual to the previous magnetic theory in the absence of the first term in the superpotential W_{mag} and without the singlets M^{ij} . It is important to note that all the gauge invariant operators discussed just above are retained except for M^{ij} even in this dual pair.

3. a -Maximization in the $Spin(10)$ Theories

When the gauge invariant chiral primary operators hit the unitarity bounds, they decouple from the remaining system as free fields of the $U(1)_R$ charge $2/3$. Therefore, following the prescription of the paper [10], one needs to improve the previous a -function $a_0(R)$ as

$$a(R) = 90 + 16F[R(\Psi)] + 10N_Q F[R(Q)] - \sum_i [F[R(\Phi_i)] - F_0],$$

where Φ_i are the gauge invariant operators hitting the unitarity bounds. Conversely, in order to find the solution to the a -maximization condition (1.1), one needs to look at all real values of x or equivalently R . As can be read from Table 1, the following unitarity bounds⁸

$$\begin{aligned}
R(M) = 2R &\geq \frac{2}{3} &\Rightarrow R &\geq \frac{1}{3}, \\
R(Y) = N_Q - 6 - (N_Q - 1)R &\geq \frac{2}{3} &\Rightarrow R &\leq \frac{1}{N_Q - 1} \left(N_Q - \frac{20}{3} \right), \\
R(B) = N_Q - 6 - (N_Q - 5)R &\geq \frac{2}{3} &\Rightarrow R &\leq \frac{1}{N_Q - 5} \left(N_Q - \frac{20}{3} \right), \\
R(D_2) = 10R &\geq \frac{2}{3} &\Rightarrow R &\geq \frac{1}{15}, \\
R(E_2) = (N_Q - 6) - (N_Q - 9)R &\geq \frac{2}{3} &\Rightarrow R &\leq \frac{1}{N_Q - 9} \left(N_Q - \frac{20}{3} \right), \\
R(D_1) = 8R + 1 &\geq 1 &\Rightarrow R &\geq 0, \\
R(E_1) = (N_Q - 5) - (N_Q - 7)R &\geq 1 &\Rightarrow R &\leq \frac{N_Q - 6}{N_Q - 7},
\end{aligned} \tag{3.1}$$

divide all real values of R into several regions⁹, as sketched for $N_Q = 7$ in Figure 2. Note that, when the denominator on the right hand side in the conditions for R from the unitarity bounds for $R(E_n)$ is zero, the corresponding unitarity bounds are independent of R and are always satisfied.

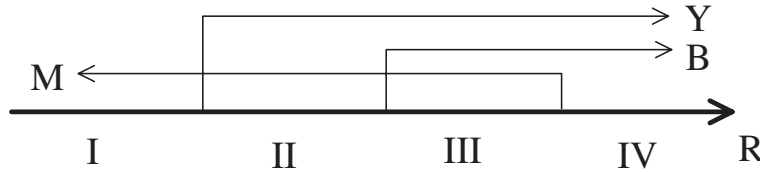


Figure 2: A sketch of operators hitting the unitarity bounds for the theory with 7 vectors. Each of the regions from I to IV are separated at $R(Q) = 1/18, 1/6, 1/3$, respectively. The arrows show the regions where the corresponding operators hit the unitarity bounds.

⁸ The unitarity bound for a spin one-half field is given [8] by $D \geq \frac{3}{2}$, which gives the bound for $U(1)_R$ charge; $R \geq 1$.

⁹ We don't take account of the unitarity bounds for the gauge invariant operators D_0 and E_0 , since they are redundant, as discussed previously.

There is a subtle point about the massless spectrum of the gauge invariant operators. The gauge invariant chiral superfields M , Y , B , D_2 , and E_2 parametrize the classical moduli space of the electric theory, and it is thus natural to consider that they are in the massless spectrum. The magnetic theory implies that the operators D_0 and E_0 are redundant by the F-term condition, as discussed before. However, there seems no compelling arguments about whether the Lorentz spinor operators D_1 and E_1 are massless or massive. If they were massless and hit the unitarity bounds, one would encounter a problem in the calculation of a trial a -function. Since they are in the spinor representation of the Lorentz group, at present we don't understand how the a -maximization method can be extended for those operators.

In fact, in some regions of the space of the $U(1)_R$ charge, the spinor operators appear to violate the unitarity bounds. Therefore, in that case, we have to assume that they are massive and don't contribute to the a -function in the infrared. We henceforth will not take account of the operators D_1 and E_1 upon the use of the a -maximization method. However, if they are actually massless, our results could not be correct in the regions where they hit the unitarity bounds given above. We will see below that the solutions to the a -maximization condition (1.1) are found in the other region, where D_1 and E_1 don't hit the bounds. Therefore, the solutions remain valid, even when they are massless.

We will demonstrate in detail the a -maximization procedure for the case of $N_Q = 7$ vectors Q^i , and then will report our results on the other value of N_Q . Before proceeding, let us make a comment on the structure of divided regions of the space of the $U(1)_R$ charge R . When one looks at the operators hitting the unitarity bounds from minus large value of R to positive large value, the order of the hitting operators on the line of R may change, as one change the number N_Q . It turns out from the unitarity bounds (3.1) that, although one needs to consider each case for $N_Q = 7, 8, 9$, one can study the other cases $N_Q \geq 10$ in a unified manner, because the order of hitting of the operators remains unchanged for the latter cases. One also finds that, for the latter cases, there is the region where none of the gauge invariant operators hit the unitarity bounds, but no such regions for the former cases of $N_Q = 7, 8, 9$.

In the case of $N_Q = 7$ vectors, there are four regions dividing the space of the $U(1)_R$ charge R , as can be seen in Table 2 and as illustrated in Figure 2. In each region, one

	Hitting Operators	Hitting Regions
<i>I</i>	M	$R \leq \frac{1}{18}$
<i>II</i>	M, Y	$\frac{1}{18} \leq R \leq \frac{1}{6}$
<i>III</i>	M, Y, B	$\frac{1}{6} \leq R \leq \frac{1}{3}$
<i>IV</i>	Y, B	$R \geq \frac{1}{3}$

Table 2: The four regions of the $U(1)_R$ charge R for $N_Q = 7$.

finds the above a -function $a(R)$ as

$$\begin{aligned}
a(R) &= a_0(R) - \frac{N_Q(N_Q + 1)}{2} (F[R(M)] - F_0), \quad \left(R \leq \frac{1}{18}\right), \\
a(R) &= a_0(R) - \frac{N_Q(N_Q + 1)}{2} (F[R(M)] - F_0) - N_Q (F[R(Y)] - F_0), \quad \left(\frac{1}{18} \leq R \leq \frac{1}{6}\right), \\
a(R) &= a_0(R) - \frac{N_Q(N_Q + 1)}{2} (F[R(M)] - F_0) - N_Q (F[R(Y)] - F_0) \\
&\quad - \frac{N_Q!}{(N_Q - 5)!5!} (F[R(B)] - F_0), \quad \left(\frac{1}{6} \leq R \leq \frac{1}{3}\right), \\
a(R) &= a_0(R) - N_Q (F[R(Y)] - F_0) - \frac{N_Q!}{(N_Q - 5)!5!} (F[R(B)] - F_0), \quad \left(R \geq \frac{1}{3}\right),
\end{aligned}$$

with $N_Q = 7$ substituted. The whole function $a(R)$ is illustrated in Figure 3, which explicitly shows that it isn't a third order polynomial of R , but gives two local minima. As can be seen in Figure 3, there is a unique local maximum, where only the mesons M^{ij} are free and the $U(1)_R$ charge gives $R = 1/30$. It is the local maximum

$$R = \frac{3N_Q^2 - 21N_Q - 12 + 2\sqrt{-(N_Q - 6)(N_Q^2 - 29N_Q + 73)}}{3(N_Q + 3)(N_Q - 1)} \quad (3.2)$$

of the function $a_0(R) - (N_Q(N_Q + 1)/2)F(2R)$ for $N_Q = 7$.

For $N_Q = 8, 9$, as can be seen in Table 3, there are four regions on the line of the $U(1)_R$ charge R , as in Figure 4. As is different from the case of $N_Q = 7$, there is no region where the three gauge invariant operators M , Y , and B hit the unitarity bounds at the same time, but appears a new region *IV*, where only the operator Y^i hits the bound. Only for $N_Q = 9$, the exotic E_2 is available, but it doesn't violate the unitarity bound over all the values of R . If the spinor exotics E_1 and D_1 were massless, our results for the regions

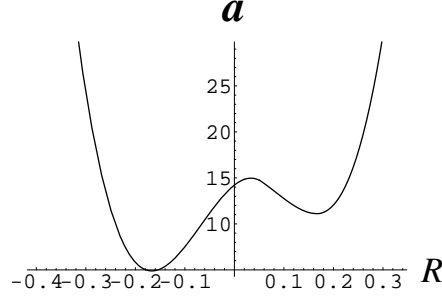


Figure 3: The whole a -function $a(R)$ for $N_Q = 7$.

	Hitting Operators	Hitting Regions
$I + II$	M	$R \leq \frac{1}{N_Q-1}(N_Q - \frac{20}{3})$
III	M, Y	$\frac{1}{N_Q-1}(N_Q - \frac{20}{3}) \leq R \leq \frac{1}{3}$
IV	Y	$\frac{1}{3} \leq R \leq \frac{1}{N_Q-5}(N_Q - \frac{20}{3})$
$V + VI$	Y, B	$R \geq \frac{1}{N_Q-5}(N_Q - \frac{20}{3})$

Table 3: The four regions of the $U(1)_R$ charge R for $N_Q = 8, 9$.

I and VI would be incomplete. The whole a -functions $a(R)$ are similar to the one for $N_Q = 7$ and has, in the region II , a single local maximum given by (3.2) with $N_Q = 8, 9$ substituted for each case, where also only the meson M^{ij} is hitting the unitarity bound to be free in the infrared. Note that the local maximum would be retained even after taking account of the exotics E_1 and D_1 .

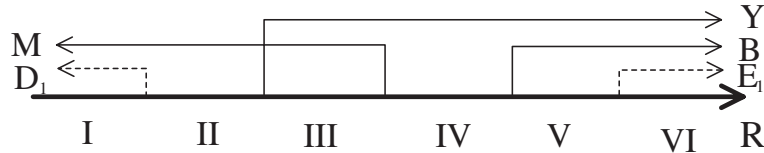


Figure 4: The operators hitting the unitarity bounds for the theory with $N_Q = 8$ and 9 vectors. The arrows show the regions where the corresponding operators hit the unitarity bounds.

For $10 \leq N_Q \leq 21$, it is remarkable that there exists the region IV with no gauge invariant operators hitting the unitarity bounds, as shown in Figure 5. The parameter space of the $U(1)_R$ charge R is divided into six regions, as can be seen in Table 4. The

	Hitting Operators	Hitting Regions
$I + II$	M, D_2	$R \leq \frac{1}{15}$
III	M	$\frac{1}{15} \leq R \leq \frac{1}{3}$
IV	no operators	$\frac{1}{3} \leq R \leq \frac{1}{N_Q-1}(N_Q - \frac{20}{3})$
V	Y	$\frac{1}{N_Q-1}(N_Q - \frac{20}{3}) \leq R \leq \frac{1}{N_Q-5}(N_Q - \frac{20}{3})$
$VI + VII$	Y, B	$\frac{1}{N_Q-5}(N_Q - \frac{20}{3}) \leq R \leq \frac{1}{N_Q-9}(N_Q - \frac{20}{3})$
$VIII$	Y, B, E_2	$R \geq \frac{1}{N_Q-9}(N_Q - \frac{20}{3})$

Table 4: The six regions on the line of the $U(1)_R$ charge R for $10 \leq N_Q \leq 21$.

regions I , VII , and $VIII$ could be incomplete due to the exotics D_1 and E_1 . The whole a -function $a(R)$ has a similar shape to the one in Figure 3. One finds a unique local maximum at

$$R = \frac{3N_Q^2 - 24N_Q - 15 + \sqrt{2885 - N_Q^2}}{3(N_Q^2 - 5)}$$

in the regions IV , where no operator hits the unitarity bound. The local maximum also remains valid even after taking account of the unitarity bounds of D_1 and E_1 .

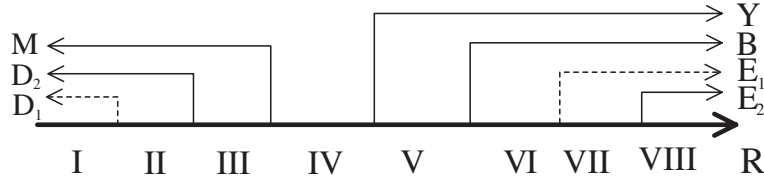


Figure 5: The operators hitting the unitarity bounds for the theory with $10 \leq N_Q \leq 21$ vectors. The arrows show the regions where the corresponding operators hit the unitarity bounds.

Finally, let us make a brief comment on the weaker version of the a -theorem [21]. From the above results, one can immediately calculate the a -function a_{IR} in the infrared and compare it to the one a_{UV} at high energy, as in Figure 6. The a -function a_{UV} counts the number of the fundamental fields as free fields, each of which contribute F_0 to it. One can see that the inequality

$$a_{\text{IR}} < a_{\text{UV}}$$

actually holds for $7 \leq N_Q \leq 21$.

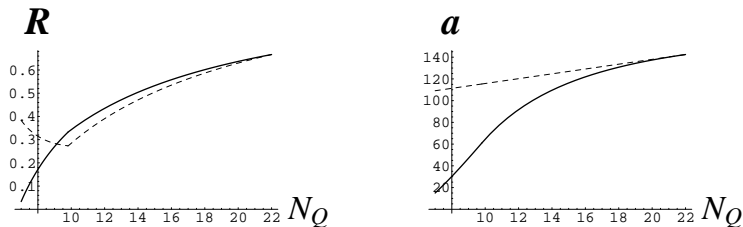


Figure 6: The graph on the left shows the R charges $R(Q)$ (solid line) and $R(\Psi)$ (dotted line). The graph on the right depicts the central charges at the UV free fixed (dotted line) and IR fixed point (solid line).

4. Summary and Discussion

We have found a unique local maximum of the a -function for $10 \leq N_Q \leq 21$, where there are no gauge invariant chiral primary operators hitting the unitarity bounds. On the other hand, for $7 \leq N_Q \leq 9$, one can also obtain a unique local maximum of the a -function, but at the local maximum, one finds that the gauge invariant operators M^{ij} are free fields at the infrared fixed point, otherwise it would violate the unitarity of the theory. The existence of the local maximum is consistent with the conjecture [3,4] that this theory is in the non-Abelian Coulomb phase for $7 \leq N_Q \leq 21$. As discussed previously, the gauge singlet spinors D_1 and E_1 could invalidate the uniqueness of the local maximum in the above mentioned regions of R . It would be interesting if we could extend the a -maximization method for such Lorentz spinor operators.

One might wonder whether the same results could be obtained in the magnetic $SU(N_Q - 5)$ theory. However, this is automatically guaranteed by the electric-magnetic duality. In fact, since the magnetic theory saturates the anomalies of all the global symmetries of the electric theory [3,4], *i.e.*, the duality satisfies the 't Hooft anomaly matching condition [15], and since the both theories give the same gauge invariant operators [3], the a -function in the magnetic theory is identical to the one in the electric theory, even when the gauge invariant operators hit the unitarity bounds.

We have found so far that the mesons M^{ij} are free in the deepest infrared for $7 \leq N_Q \leq 9$. As discussed for the $Spin(7)$ theory in Introduction, it implies that the parameter \tilde{h} in the superpotential W_{mag} of the magnetic theory goes to zero, as one goes to the infrared, as can be seen from (2.1). Since the equations of motion gives

$$\frac{\partial}{\partial M^{ij}} W_{\text{mag}} = \frac{\tilde{h}}{\tilde{\mu}^2} N_{ij} = 0,$$

where $N_{ij} = \bar{q}_i s \bar{q}_j$, if \tilde{h} weren't zero, the gauge invariant operators N_{ij} would be redundant. This is indeed the case for $10 \leq N_Q \leq 21$. However, for $7 \leq N_Q \leq 9$, since \tilde{h} goes to zero¹⁰, the operators N_{ij} don't have to be redundant. Therefore, N_{ij} should be new degrees of freedom. In fact, at the infrared fixed point, the term $M^{ij} \bar{q}_i s \bar{q}_j$ can be turned on as a perturbation in the superpotential W_{mag} with vanishing \tilde{h} . At the point, the $U(1)_R$ charge of the perturbation $M^{ij} \bar{q}_i s \bar{q}_j$ exceed two, since the $U(1)_R$ charges of M^{ij} and $\bar{q}_i s \bar{q}_j$ are equal to $2/3$ and $2 - 2R(Q) \geq 4/3$, respectively. Therefore, we can see that it is an irrelevant operator at the infrared fixed point. This is consistent with the fact that the parameter \tilde{h} goes to zero in the infrared.

Furthermore, one can easily see that the magnetic theory with vanishing \tilde{h} in the superpotential (2.1) is dual to the same $Spin(10)$ theory but with the superpotential

$$W_{\text{ele}} = N_{ij} Q^i Q^j,$$

with the gauge singlets N_{ij} and the free singlets M^{ij} . The singlets N_{ij} can be identified with $\bar{q}_i s \bar{q}_j$. Therefore, the magnetic theory of the original dual pair flows into the magnetic one of another dual pair at the infrared fixed point. It suggests that the original electric theory flows into the electric theory with the superpotential W_{ele} .

In the electric theory with the superpotential W_{ele} , we can perform the a -maximization procedure in a similar way to what we have done in this paper. The region where no gauge invariant operators hit the unitarity bounds is identical on the line of the $U(1)_R$ charge R to the region where only the operators M^{ij} hit the unitarity bound in the original electric theory. In the region, the trial a -function can be calculated in terms of the fundamental fields at high energy in the former theory to give

$$a_0(R) + \frac{N_Q(N_Q + 1)}{2} F[R(N)] + \frac{N_Q(N_Q + 1)}{2} F_0,$$

where $a_0(R)$ is given in (2.2), and F_0 is the contribution from the free singlets M^{ij} . Since the function $F(x)$ satisfies the relation

$$F(x) + F(2 - x) = 0, \tag{4.1}$$

one notices that $F[R(N)] = -F[R(QQ)]$ and that the above a -function is the same as the one in the identical region in the original electric theory. Since the latter a -function

¹⁰ For $N_Q = 7$, the parameter \tilde{h}'' also goes to zero.

are constructed via the prescription of [10], one find that it is consistent with the electric-magnetic duality.

As argued in Introduction, the auxiliary field method can be applied to the theories under consideration. In the original theory, turning on the superpotential

$$W = N_{ij} (Q^i Q^j - h M^{ij}),$$

to introduce the auxiliary fields M^{ij} with the Lagrange multipliers N_{ij} , one can easily conceive that the parameter h goes to zero in the infrared, due to the consistency with the result that the singlets M^{ij} go to free fields in the magnetic theory for $7 \leq N_Q \leq 9$. Furthermore, when h goes to zero, one obtains the superpotential W_{ele} of the other electric theory introduced above. It means that the original electric theory flows into the other electric theory with W_{ele} and thus is consistent with the magnetic picture. The equation of motion from the superpotential W_{ele} yields the constraint

$$\frac{\partial}{\partial N_{ij}} W_{\text{ele}} = Q^i Q^j = 0.$$

It is also consistent with the result that the composites M^{ij} decouple from the remaining system in the original theory.

One may raise a question whether the auxiliary field method affects our results via a -maximization in the last section, because we introduced the auxiliary fields M^{ij} and the Lagrange multipliers N_{ij} charged under $U(1) \times U(1)_R$. This is however not the case, since as has been discussed in [11], the massive fields M^{ij} and N_{ij} don't contribute to the a -function, due to (4.1). But, once the singlet M^{ij} hits the unitarity bound, an accidental $U(1)_M$ symmetry appears to fix the $U(1)_R$ charge of M^{ij} to $2/3$. On the other hand, the singlets N_{ij} are still interacting with the vectors Q^i in the superpotential, and their $U(1)_R$ charge remains unchanged and contributes $F[2 - R(Q^i Q_j)] = -F[2R(Q)]$ to the a -function;

$$F[R(M)] + F[R(N)] \Rightarrow F(2/3) + F[2 - R(Q^i Q_j)] = -F[2R(Q)] + F_0.$$

One can thus see that it gives the identical procedure to what we have done in the previous section when the meson M^{ij} hits the unitarity bound.

The auxiliary field method plausibly seems to work well to describe the flow of the original electric theory into the other electric one. We have seen that it gives a quite consistent picture with the magnetic one. However, if it is true, it seems to suggest a

striking mechanism, because it means that the mass term $hN_{ij}M^{ij}$ goes to zero at the infrared fixed point. It is rather counter-intuitive and also is against *naturalness*. It would be interesting to inquire whether it could be a solution to the μ problem in the supersymmetric standard model¹¹. Since the deeper implication of it is beyond the scope of this paper, we will have to leave it to the future.

Although, in this paper, we restrict ourselves to the $Spin(10)$ gauge theory with a single spinor and N_Q vectors, one can immediately extend it for the one with more than one spinor and N_Q vectors [23]. We will report the results about it in another paper [18], and we will also there give a detailed study of the deformations and the Higgs effect via a -maximization in the model discussed in this paper, as in [24] for the model discussed in [10,25].

Note added: after submission to the arXive, we notice that the gauge invariant operator $E_{1\alpha}$ is also redundant. Therefore, it doesn't invalidate our results in the region VI for $N_Q = 8, 9$ and in the regions VII and $VIII$ for $10 \leq N_Q \leq 21$. In order to prove that $E_{1\alpha}$ is redundant, as is discussed in [26,27], we need to notice that, in the magnetic theory

$$\bar{q}_j W_\alpha = \frac{1}{4} \bar{D}^2 [D_\alpha (\bar{q}_j e^{-V}) e^V],$$

where $W_\alpha = (1/4) \bar{D}^2 [D_\alpha e^{-V} \cdot e^V]$ and V is the vector superfield of the dual gauge group $SU(N_Q - 5)$. Making use of the equation of motion $\partial W_{\text{mag}} / \partial s = 0$, we find that

$$\begin{aligned} (*E_{1\alpha})_{j_1 \dots j_{N_Q-7}} &\sim \tilde{h} \tilde{\mu}^{N_Q-10} M^{kl} \bar{q}_k \bar{q}_{j_1} \dots \bar{q}_{j_{N_Q-7}} \left(\bar{q}_l \tilde{W}_\alpha \right) \\ &\sim \frac{1}{4} \bar{D}^2 \left[\tilde{h} \tilde{\mu}^{N_Q-10} M^{kl} \bar{q}_k \bar{q}_{j_1} \dots \bar{q}_{j_{N_Q-7}} D_\alpha (\bar{q}_l e^{-V}) e^V \right]. \end{aligned}$$

Thus, the Lorentz spinor $E_{1\alpha}$ is redundant.

¹¹ For a recent study of another phenomenological topic of this model, see [22].

Acknowledgements

We wish to acknowledge discussions with Masahiro Ibe, Takuya Okuda, and Yuuki Shinbara and to especially thank Yu Nakayama for helpful discussions on a -maximization and quantum moduli spaces. We are also grateful to Kenneth Intriligator for kind correspondence. T. K. and Y. O. would like to thank the participants of the Summer Institute String Theory 2005 at Sapporo, Japan for discussions. Y. O. and F. Y. thank the Yukawa Institute for Theoretical Physics at Kyoto University. Discussions during the YITP workshop YITP-W-05-08 on String Theory and Quantum Field Theory were useful to complete this work. The research of T. K. was supported in part by the Grants-in-Aid (#16740133) and (#16081206) from the Ministry of Education, Science, Sports, and Culture of Japan. The research of Y. O. was supported in part by JSPS Research Fellowships for Young Scientists. The research of Y. T. was supported in part by the National Science Foundation under Grant No. PHY99-07949 and in part by JSPS Research Fellowships for Young Scientists.

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